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The type-I D-instanton and its M-theory origin

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Abstract: The tree-level amplitude for the scattering of two gauge particles constrained to move on the two distinct boundaries of eleven-dimensional space-time in the Hořava-Witten formulation of M-theory is constructed. At low momenta this reproduces the corresponding tree-level scattering amplitude of the $E_8 \times E_8$ heterotic string theory. After compactification to nine dimensions on a large circle with a suitable Wilson line to break the symmetry to SO(16) $\times$ SO(16) this amplitude is used to describe the scattering of two massive SO(16) spinor states — one from each factor of the unbroken symmetry group. The amplitude contains a component that is associated with the exchange of a Kaluza-Klein charge between the boundaries, which is interpreted as the exchange of a D-particle between orientifold planes in the type-IA theory. This is related by T-duality to the effect of a non-BPS D-instanton in the type-I theory which is only invariant under those elements of O(16) $\times$ SO(16) that are in SO(16) $\times$ SO(16).

Keywords: Superstrings and Heterotic Strings, M-Theory, D-branes, String Duality.

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Contents

1. Introduction

Over the past two years there has been a great deal of progress in the study of non-BPS solitonic states in string theory [1]. Among these are the stable non-BPS D-branes of certain orbifold theories [2]–[5], and of the type-I (and type-IA) theories [6]–[10]. All of these stable non-BPS states can be elegantly characterized in terms of K-theory [11].

Most of the above non-BPS D-branes can be obtained by suitable projections from the corresponding unstable non-BPS states of the type-II theories [12]. These unstable non-BPS Dp-branes occur for the complementary values of the (stable) BPS branes, i.e. they have odd p in type IIA and even p in type IIB. However, in the case of type I there exists also a $\mathbb{Z}_2$ D-instanton (D(-1)-brane), which has the same value of p as the corresponding type-IIB D(-1)-brane. (The same is also the case for the type-I D7-brane.) The presence of the type-I D-instanton is associated with the fact that the actual gauge group of type I is SO(32) rather than the gauge group of the perturbative type-I theory which is O(32).

In this paper we will determine some explicit effects of the type-I D-instanton by studying a simple scattering process in M-theory that can be identified, via familiar dualities, with a type-I string theory process. We will start, in section 2, by consider-

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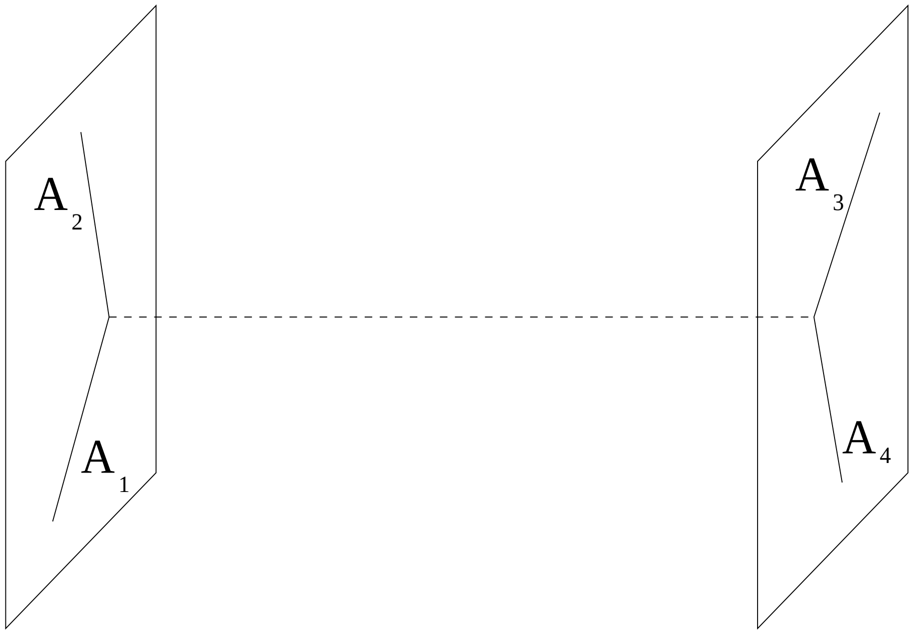

Figure 1: Tree scattering diagram of four gauge particles

ing dynamics in the eleven-dimensional geometry considered by Hořava and Witten, in which the eleven-dimensional M-theory space-time is taken to be $M^{10} \times R$, where M^{10} is ten-dimensional Minkowski space and R is the interval $0 \leq x^{11} \leq d$ [13, 14]. The tree-level contribution to the scattering of two E_8 gauge particles that are confined to distinct boundaries of the eleven-dimensional space-time will be evaluated in section 3, as illustrated in figure 1. Since the external particles are localized on distinct boundaries the exchanged bulk states are singlets under both the E_8 gauge groups. We will verify that in the low-momentum limit this amplitude reproduces the same expression as that obtained in the $E_{8\,L} \times E_{8\,R}$ heterotic string (which will be referred to as the HE theory) in the same limit, where the subscripts L and R refer to the two boundaries (left and right) at $x^{11} = 0$ and $x^{11} = d$. This is true independent of the separation of the boundaries.

In section 4 we will consider the theory compactified on a circle, S^1, in the x^9 direction with the symmetry broken to $\mathrm{SO}(16)_L \times \mathrm{SO}(16)_R$. We will be particularly interested in the scattering of massive spinor states localized on the two nine-dimensional boundaries that are modes of the massless ten-dimensional states with Kaluza-Klein charge $\pm 1/2$. The four-point function for these states follows very simply from the ten-dimensional expression. We will also describe the interpretation of this amplitude in terms of the type-IA theory in which the background is that of the type-IIA theory on the $\mathbb{Z}_2$ orbifold of a circle. In this description the scattering spinor states are D-particles (D_S) or anti D-particles ($\bar{D}_S$) stuck to the two orientifold planes. The scattering amplitudes that will be considered are "tree-level" interactions. Processes of the form $D_S + D_S \to D_S + D_S$, $D_S + \bar{D}_S \to D_S + \bar{D}_S$

are described by the exchange of closed strings (states of zero Kaluza-Klein charge) between the separated orientifold planes.[1] At low energies the former process is BPS, and the amplitude vanishes in the zero velocity limit. In the impact parameter description, the resulting force is proportional to v^4 which is in agreement with the string theory calculation that can be performed following [15]. The second process is non BPS, and the amplitude is singular as $v \to 0$, reflecting the instability of the system to decay into a non-BPS D1-brane.

The process $D_S + \bar{D}_S \to \bar{D}_S + D_S$ is described by the exchange of a D0-brane (the state of Kaluza-Klein charge 1) between the separated orientifold planes. The effect of a massive particle exchange generates an exponentially small amplitude. After a T-duality in the x^{11} direction this is reinterpreted as the effect of the type-I D-instanton. This is analogous to the description of the type-IIB D-instanton as the dual of the world-line of a type-IIA D-particle [16] which also has an origin in eleven-dimensional supergravity [17]. As in that case, the supergravity Feynman diagram calculation automatically takes into account the appropriate quantum measure, including fermionic zero modes. These modes are interpreted in terms of massless string states in the type-IA picture. As expected, the D-instanton contribution respects $\mathrm{SO}(16)_L \times \mathrm{SO}(16)_R$ but is not invariant under $\mathrm{O}(16)_L \times \mathrm{SO}(16)_R$ or $\mathrm{SO}(16)_L \times \mathrm{O}(16)_R$, as will be described in some detail in section 4.2. Finally, the discussion in section 5 contains some speculations about the situation in which the four particles in the scattering amplitude are located on one of the two boundaries in the nine-dimensional theory.

2. Feynman diagrams in the Hořava-Witten geometry

We are interested in calculating the tree-level scattering amplitude illustrated in figure 1, in which the propagator joins the two boundaries which are separated by a distance d. The scattering particles with momenta k_r $(r = 1, \ldots, 4)$ are the states confined to the ten-dimensional boundaries which are massless E_8 gauge particles in the unbroken theory. Later we will consider compactification on a circle, in which case the gauge group may be broken and the scattering particles may carry non-zero Kaluza-Klein charge and be massive states. When the external particles are bosons (as will be the case in this paper) the propagating intermediate state in the figure may be either a graviton, $h_{\mu\nu}$, or a third-rank antisymmetric tensor potential, $C^{(3)}_{\mu\nu\rho}$, and the full amplitude is obtained by summing over both these contributions (where the eleven-dimensional indices span the range $\mu, \nu, \rho = 0, 1, \ldots, 9, 11$).[2]

[1] Here the notation $L + R \to L' + R'$ denotes a scattering process in which L and R are incoming states on the left and right orientifolds, respectively, while the corresponding outgoing states are L' and R'.

[2] Recently a similar calculation was undertaken by Krause [18] but the three-form exchange was omitted so his result is therefore not complete.

The Hořava-Witten geometry may be thought of as a $\mathbb{Z}_2$ orbifold, where the generator of the orbifold group acts on the eleventh dimension, x^{11}, as $x^{11} \mapsto -x^{11}$, as well as acting on the three-form field C as $C \mapsto -C$. In the resulting theory the momentum in the eleventh dimension, p_{11}, is not conserved, whilst it is still conserved in the tangential ten dimensions. Furthermore, the boundary conditions on the bulk fields require that $h_{M\,11} = 0 = C_{MNP}$ on the boundaries $x^{11} = 0$ and $x^{11} = d$, where $M, N, P = 0, 1, \dots, 9$ are the ten-dimensional indices.[3]

We will consider the simple example of the propagator for a scalar field in this background by first considering M-theory compactified on a circle with background geometry $M_{10} \times S^1$, where S^1 is a circle of radius $d/\pi = R_{11} l_p$ (where R_{11} is the dimensionless radius and l_p is the eleven-dimensional Planck length). The momentum-space scalar propagator is

$$\tilde{G}(p, p_{11}) = \frac{1}{p^2 + p_{11}^2}\,, \tag{2.1}$$

where $p_{11} = m(R_{11} l_p)^{-1} = \pi m d^{-1}$ and p^M $(M = 0, 1, \dots, 9)$ is the arbitrary momentum in the ten dimensions tangential to the boundaries. The scalar propagator evaluated between two points, x^{11} and y^{11}, on the circle is therefore given by the Fourier sum,

$$G(p^M; x^{11} - y^{11}) = \frac{1}{2d} \sum_{m=-\infty}^{\infty} e^{i\frac{\pi m}{d}(x^{11} - y^{11})} \frac{1}{p^2 + \pi^2 m^2/d^2}\,. \tag{2.2}$$

We are really interested in defining the propagator on the $\mathbb{Z}_2$ orbifold of the circle. If we choose the scalar field to be symmetric under the action of the $\mathbb{Z}_2$ then the propagator must be invariant under $x^{11} \mapsto -x^{11}$ as well as $x^{11} \mapsto x^{11} + 2d$ (with similar conditions on y^{11}). The propagator satisfying these conditions is then given by

$$\mathcal{G}\left(p^M; x^{11}, y^{11}\right) = G\left(p^M; x^{11} - y^{11}\right) + G\left(p^M; x^{11} + y^{11}\right). \tag{2.3}$$

Since the external particles are constrained to the planes $x^{11} = 0$ and $x^{11} = d$, the propagator that enters in the scattering amplitude is obtained by substituting (2.2) into (2.3) and setting $x^{11} = 0$, $y^{11} = d$, which gives,

$$\begin{aligned} \mathcal{G}(p^M; 0, d) &= \frac{1}{d} \sum_{m=-\infty}^{\infty} \frac{(-1)^m}{p^2 + \pi^2 m^2/d^2} = \frac{1}{d} \sum_{m=-\infty}^{\infty} (-1)^m \int_0^\infty d\sigma\, e^{-\sigma(p^2 + \pi^2 m^2/d^2)}\,, \\ &= \frac{1}{\sqrt{\pi}} \sum_{n=-\infty}^{\infty} \int_0^\infty d\sigma\, \sigma^{-1/2} e^{-\sigma p^2 - \frac{d^2}{\sigma}(n+1/2)^2}\,, \end{aligned} \tag{2.4}$$

where the last line follows after performing a Poisson resummation that replaces the integer Kaluza-Klein charge m by the integer $2n + 1$ that labels the winding number of the propagator around the x^{11} direction. (In particular, the sum in (2.4)

[3] We are here following the "upstairs" formalism of [14], in which the fields span the covering space $\mathbb{R}^{1,9} \times S^1$ and the orbifold conditions are imposed by modding out by the action of $\mathbb{Z}_2$.

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only contains contributions with odd winding number; this reflects the fact that the states in the propagator run from $x^{11} = 0$ to $x^{11} = d$.) The σ integral is standard (the saddle point approximation is exact) and leads to the result

$$\mathcal{G}(p^M; 0, d) = \frac{1}{\sqrt{p^2}} \sum_{n=-\infty}^{\infty} e^{-|2n+1|\sqrt{p^2} d} = \frac{1}{\sqrt{p^2}} \frac{1}{\sinh(\sqrt{p^2} d)} \,. \tag{2.5}$$

In deriving (2.5) we have assumed that p^2 is positive (so that the integral in (2.4) converges). However it is clear from the definition of $\mathcal{G}(p^M; 0, d)$ in (2.4) that $\mathcal{G}$ is a meromorphic function of p^2; since the r.h.s. of (2.5) is also meromorphic, the final result will therefore hold in general (i.e. not only for positive p^2). We will see later how the terms in the sum over n correspond to the contributions of D-instantons to the type-I theory in nine dimensions.

In the amplitude calculations that follow, the vertices will involve momentum factors. In the case of the exchange of the three-form potential this will result in a term in which a factor of p_{11}^2 is inserted into the numerator of the propagator, so that the sum in (2.4) is replaced by

$$\begin{aligned} \hat{\mathcal{G}}\left(p^M; 0, d\right) &= \frac{1}{d} \sum_{m=-\infty}^{\infty} \frac{(-1)^m \, p_{11}^2}{p^2 + p_{11}^2} = -\frac{1}{d} \sum_{m \in \mathbb{Z}} (-1)^m \int_0^\infty d\sigma e^{-\sigma p^2} \frac{d}{d\sigma} e^{-\frac{\pi^2 m^2}{d^2}\sigma} \\ &= -\frac{1}{\sqrt{\pi}} \sum_{n \in \mathbb{Z}} \int_0^\infty d\sigma \; e^{-\sigma p^2} \frac{d}{d\sigma} \left(\sigma^{-\frac{1}{2}} e^{-\frac{d^2}{\sigma}(n+1/2)^2} \right) \\ &= -\sqrt{p^2} \frac{1}{\sinh(\sqrt{p^2} d)} \,, \end{aligned} \tag{2.6}$$

where a Poisson resummation has been performed in the second step, and we have integrated by parts and used (2.5) to obtain the final line.

The propagators that enter into the tree diagrams of interest are those of the metric and the three-form potential. Writing the metric in non-compact eleven-dimensional space-time as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $\eta_{\mu\nu}$ is the eleven-dimensional background Minkowski metric, the $\langle h_{\mu_1\nu_1} h_{\mu_2\nu_2} \rangle$ propagator (in the de Donder gauge) is given by

$$\tilde{G}_{\mu_1\nu_1;\mu_2\nu_2}(p^M, p_{11}) = \kappa^2 \left(\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2} + \eta_{\mu_1\nu_2}\eta_{\nu_1\mu_2} - \frac{2}{9}\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2} \right) \frac{1}{p^2 + p_{11}^2} \,. \tag{2.7}$$

Similarly, writing $C^{(3)}_{\mu\nu\rho} = c^{(3)}_{\mu\nu\rho} + \hat{C}^{(3)}_{\mu\nu\rho}$, where $c^{(3)}_{\mu\nu\rho} = 0$ is the background antisymmetric potential, the $\langle \hat{C}^{(3)}_{\mu_1\nu_1\rho_1} \hat{C}^{(3)}_{\mu_2\nu_2\rho_2} \rangle$ propagator is given (in Feynman gauge) by

$$\tilde{G}_{\mu_1\nu_1\rho_1;\mu_2\nu_2\rho_2}\left(p^M, p_{11}\right) = \frac{\kappa^2}{(3!)^2} \eta_{\mu_1[\mu_2}\eta_{|\nu_1|\nu_2}\eta_{|\rho_1|\rho_2]} \frac{1}{p^2 + p_{11}^2} \,. \tag{2.8}$$

The vertices that couple these propagators to the gauge particles in the boundaries are determined by the supergravity action in the Hořava-Witten geometry which is given in [14] and reproduced in appendix A. The tree-level amplitudes involve the cubic vertices coupling a pair of gauge particles to a graviton or an antisymmetric potential. The graviton coupling AAh is given by

$$\begin{aligned} S_{YM}^{AAh} = & \frac{1}{(4\pi)^{5/3}\kappa^{4/3}} \int_{\mathbb{R}^{1,9}} d^{10}x \left(\partial_M A_N^a \partial_P A_Q^a h_{RS}\right) \times \\ & \times \left(\frac{1}{2}\eta^{M[Q}\eta^{P]N}\eta^{RS} + \eta^{M[P}\eta^{Q]S}\eta^{RN} + \eta^{M[R}\eta^{Q]N}\eta^{PS}\right). \end{aligned} \tag{2.9}$$

The three-form coupling $AA\hat{C}$ is extracted from the G^2 term in the bulk action. This interaction arises because the Bianchi identity for the bulk field strength $G_{MNP\,11}$ receives a boundary contribution in a manner that is determined by requiring local supersymmetry as discussed in [14],

$$G_{MNP\,11} = 4!\partial_{[M}\hat{C}_{NP\,11]} + \frac{\kappa^{2/3}}{\sqrt{2}(4\pi)^{5/3}} \left(\delta(x^{11}) + \delta(x^{11}-d)\right)\omega_{MNP}\,, \tag{2.10}$$

where ω is the Chern-Simons three-form defined by

$$\omega_{MNP} = 2\,\mathrm{Tr}\left(A_M\partial_{[N}A_{P]} + \frac{1}{3}A_M[A_N, A_P] + \text{cyclic perms.}\right). \tag{2.11}$$

The other components of G are eliminated on the boundary since the three-form field is anti-symmetric under $x^{11} \mapsto -x^{11}$. The AAC interaction comes from the term linear in ω_{MNP} in $G^2_{MNP\,11}$ and is given by

$$\begin{aligned} S^{AAC} = & \frac{\sqrt{2}}{(4\pi)^{5/3}\kappa^{4/3}} \int_{\mathbb{R}^{1,9}\times S^1} d^{11}x \left(\delta(x^{11}) + \delta(x^{11}-d)\right) \times \\ & \times \partial_{[M}\hat{C}_{NP11]}\,\mathrm{Tr}\left(A^M\partial^{[N}A^{P]}\right). \end{aligned} \tag{2.12}$$

3. Scattering of boundary gauge particles

We will now consider the amplitude illustrated in figure 1 that describes tree-level elastic scattering of an incoming gauge particle localized on one of the boundaries with a gauge particle localized on the other boundary. Since the exchanged bulk fields do not carry gauge quantum numbers the diagram has an overall group theory factor $\mathrm{Tr}_L(T_1T_2)\,\mathrm{Tr}_R(T_3T_4)$, where Tr_L denotes the trace over the 248 components of the adjoint representation of the $E_{8\,L}$ gauge group on one boundary while Tr_R denotes the trace over $E_{8\,R}$ on the other. We will verify that this amplitude reduces in the limit of small momenta to the same expression as that of the tree-level $E_{8\,L} \times E_{8\,R}$

heterotic string with coupling constant related to the radius R_{11} in the usual manner. We will only consider the case in which all the external states are gauge bosons, in which case the exchanged states are the three-form potential and the graviton.

The momentum of the scattering states is restricted to the ten flat directions parallel to the boundaries, and the Mandelstam variables s, t, u are defined in terms of the ten-dimensional momenta k_1, k_2, k_3, k_4 by

$$s = -(k_1 + k_2)^2 \,, \qquad t = -(k_1 + k_4)^2 \,, \qquad u = -(k_1 + k_3)^2 \,, \tag{3.1}$$

with $s + t + u = 0$, which follows from momentum conservation, $\sum_r k_r = 0$, and the mass-shell conditions, $k_r^2 = 0$, for the external physical states. These vector states have polarization vectors ϵ_r^M which satisfy the transversality conditions $\epsilon_r \cdot k_r = 0$.

The non-standard aspect of the tree diagrams illustrated in figure 1 is that the two vertices are constrained to lie in the boundaries at $x^{11} = 0$ and $x^{11} = d = \pi R_{11} l_p$. This means that momentum is not conserved in the eleventh direction and we must use propagators of the form $\mathcal{G}(p; 0, d)$, suitably generalized to account for the tensor indices on the propagating fields as in (2.7) and (2.8).

The expressions for the tree amplitude due to the graviton and the three-form exchange are given in appendix B by $A_4^{(h)}$ (B.2) and $A_4^{(C)}$ (B.3). The complete tree-level contribution is the sum of these two amplitudes, and it is given by

$$A_4 = \frac{\kappa^{-2/3}}{4(4\pi)^{10/3}} \,\mathrm{Tr}(T_1 T_2)\,\mathrm{Tr}(T_3 T_4) \frac{1}{\sqrt{-s}} \frac{1}{\sinh(\sqrt{-s}d)} t_8 F^4 \,, \tag{3.2}$$

where

$$\begin{aligned} t_8 F^4 = \Big\{ &- 2ut(\epsilon_1 \cdot \epsilon_2)(\epsilon_3 \cdot \epsilon_4) - 2st(\epsilon_1 \cdot \epsilon_3)(\epsilon_2 \cdot \epsilon_4) - 2su(\epsilon_1 \cdot \epsilon_4)(\epsilon_2 \cdot \epsilon_3) + \\ &+ (\epsilon_1 \cdot \epsilon_2)\Big[4t(\epsilon_3 \cdot k_1)(\epsilon_4 \cdot k_2) + 4u(\epsilon_3 \cdot k_2)(\epsilon_4 \cdot k_1)\Big] + \\ &+ (\epsilon_3 \cdot \epsilon_4)\Big[4t(\epsilon_1 \cdot k_3)(\epsilon_2 \cdot k_4) + 4u(\epsilon_1 \cdot k_4)(\epsilon_2 \cdot k_3)\Big] + \\ &+ (\epsilon_1 \cdot \epsilon_3)\Big[4s(\epsilon_2 \cdot k_3)(\epsilon_4 \cdot k_1) + 4t(\epsilon_2 \cdot k_1)(\epsilon_4 \cdot k_3)\Big] + \\ &+ (\epsilon_2 \cdot \epsilon_4)\Big[4s(\epsilon_1 \cdot k_4)(\epsilon_3 \cdot k_2) + 4t(\epsilon_1 \cdot k_2)(\epsilon_3 \cdot k_4)\Big] + \\ &+ (\epsilon_1 \cdot \epsilon_4)\Big[4s(\epsilon_2 \cdot k_4)(\epsilon_3 \cdot k_1) + 4u(\epsilon_2 \cdot k_1)(\epsilon_3 \cdot k_4)\Big] + \\ &+ (\epsilon_2 \cdot \epsilon_3)\Big[4s(\epsilon_1 \cdot k_3)(\epsilon_4 \cdot k_2) + 4u(\epsilon_1 \cdot k_2)(\epsilon_4 \cdot k_3)\Big]\Big\} \,. \end{aligned} \tag{3.3}$$

In the limit of low momenta (i.e. s, t, u small) the above amplitude becomes

$$A_4 \sim -\frac{\kappa^{-2/3}}{4(4\pi)^{10/3}} \,\mathrm{Tr}(T_1 T_2)\,\mathrm{Tr}(T_3 T_4) \frac{1}{sd} t_8 F^4 \,. \tag{3.4}$$

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In order to compare this to the heterotic string we need to rewrite

$$\frac{\kappa^{-2/3}}{d} = \frac{\kappa^{-2/3}}{\pi R_{11} l_p} = \frac{(2\pi l_p)^{-3}}{\pi R_{11} l_p} = \frac{2}{(2\pi l_p)^4 R_{11}} \tag{3.5}$$

in string units, using the standard relation between l_p and the heterotic string length scale, l_s^H,

$$l_p^2 = R_{11} \left(l_s^H\right)^2 . \tag{3.6}$$

We also need the relation between R_{11} and the coupling constant, $g_E = e^{\phi^E}$, of the ten-dimensional heterotic $E_8 \times E_8$ (HE) string theory [13],

$$R_{11} = g_E^{2/3} = e^{2\phi^E/3} . \tag{3.7}$$

The amplitude in heterotic string units is then given by

$$A_4^{HE} \sim -(2\pi l_s^H)^{-4} e^{-2\phi^E} \operatorname{Tr}(T_1 T_2) \operatorname{Tr}(T_3 T_4) \frac{1}{s} t_8 F^4, \tag{3.8}$$

which agrees with the tree-level heterotic result in the low-momentum limit [21, 22].

This agreement should not seem surprising in view of the fact that the low-momentum tree-level heterotic string amplitude is equal to the tree-level amplitude of ten-dimensional $N = 1$ supergravity coupled to $E_{8\,L} \times E_{8\,R}$ $N = 1$ supersymmetric Yang-Mills theory. In the small-R_{11} limit (the heterotic string weak coupling limit) the two Hořava-Witten boundaries coincide and our tree-level field theory calculation becomes ten dimensional. The slightly subtle point is that in the Hořava-Witten case the full tree amplitude is given by the sum of "pole diagrams" which only involve three-point vertices while the ten-dimensional supergravity calculation also involves the contribution of a "contact" interaction of the form $\omega_{MNP}\omega^{MNP}$ (the four gauge particle contact term does not contribute to the particular group theory factor that is of interest to us). In fact, it is easy to see that this contact term is reproduced precisely by the second contribution $A_4^{(C)\,2}$ in appendix B.

4. Compactification on S^1

Upon compactification on a circle of radius $R_9 l_p$ in the x^9 direction the gauge group may be broken by the introduction of Wilson lines. We will here consider the case in which the unbroken group is $\mathrm{SO}(16)_L \times \mathrm{SO}(16)_R$. The two nine-dimensional heterotic strings with this gauge group are T-dual to each other. This system has a description in type-IA language in which the Hořava-Witten boundaries are represented by two orientifold eight-planes separated by $\pi R_{11} l_p$ with eight D8-branes and their images placed coincident with each of them.

The adjoint representation of E_8 decomposes into the adjoint and one of the two chiral spinor representations of the SO(16) subgroup, $\mathbf{248} = \mathbf{120}+\mathbf{128}$. The breaking of the E_8 symmetry is accompanied by the generation of a mass for the chiral spinor

128 which is a state with Kaluza-Klein charge $l = \pm 1/2, \pm 3/2, \ldots$. The chirality of these states is independent of the signs of the charges. In eleven-dimensional units, the mass of the spinor states is given by

$$M_S = \frac{|l|}{R_9 l_p}\,. \tag{4.1}$$

We will mainly consider external states that are of lowest mass, and therefore have $l = \pm 1/2$. We shall also choose to satisfy $M_S \ll M_{\rm planck}$; in this case we may define a "low energy" limiting effective field theory in which the spinor states survive but Planck-scale or string-scale excitations can be ignored. In the type-IA description a spinor state corresponds to a single D-particle or anti D-particle stuck on an orientifold planes. In terms of the eleven-dimensional moduli, the type-IA string coupling constant $g_{\rm IA}$ is given as [13],

$$g_{\rm IA} = R_9^{3/2}\,, \tag{4.2}$$

and the type-I string length, $l_s^{\rm I}$, is given in terms of the Planck length by

$$\frac{l_s^{\rm I}}{l_p} = g_{\rm IA}^{-1/3} = R_9^{-1/2}\,. \tag{4.3}$$

Thus in terms of the type-IA theory, the mass M_S for $l = \pm 1/2$ becomes

$$M_S = \frac{1}{2R_9 l_p} = \frac{1}{l_s^{\rm I}} \frac{1}{2g_{\rm IA}}\,, \tag{4.4}$$

which is indeed the mass of a stuck D-particle (i.e. of a D_S).

4.1 The different kinematical regimes

Since the SO(16) spinor states carry Kaluza-Klein charge $l = \pm 1/2$ there are three distinct classes of four-point functions in the nine-dimensional theory. These are characterized by which of the incoming and outgoing particles are D_S and which are $\bar{D}_S$. We are considering the scattering process as a T-channel process in which K_1 and K_4 are the nine-dimensional momenta of the incoming particles that scatter into outgoing particles with momenta $-K_2$ and $-K_3$. These processes are the following:

(a) $D_S + D_S \to D_S + D_S$, which has total S-channel Kaluza-Klein charge $l = 0$. At low velocity this becomes a BPS configuration that preserves supersymmetry.

(b) $D_S + \bar{D}_S \to D_S + \bar{D}_S$ also has $l = 0$ but is far from BPS at low velocity.

(c) $D_S + \bar{D}_S \to \bar{D}_S + D_S$ has $l = 1$. Although this is far from BPS at low T-channel velocity the S-channel process that is related by crossing symmetry is BPS at low S-channel velocity. In this process a bulk D0-brane with mass $2M_S$ is exchanged.

The processes obtained by interchanging all D_S's with $\bar{D}_S$'s are trivially related to these three processes and need not be considered separately.

We will describe the kinematics in the nine-dimensional centre of mass frame with the outgoing states scattering through an angle θ relative to the incoming states. Writing the ten-dimensional momentum of the r-th particle as $k_r = (K_r, q_9)$, where q_9 is the Kaluza-Klein momentum in the circular x^9 direction, gives the explicit expressions

$$\begin{aligned} k_1 &= (E, \mathbf{0}, 0, p, M_S) \equiv (K_1, M_S)\,, & (4.5)\\ k_2 &= \left(-E', \mathbf{0}, -p\sin\theta, -p\cos\theta, -(-1)^l M_S\right) \equiv (K_2, -(-1)^l M_S)\,, & (4.6)\\ k_3 &= \left(-E', \mathbf{0}, p\sin\theta, p\cos\theta, -(-1)^{l+r} M_S\right) \equiv (K_3, -(-1)^{l+r} M_S)\,, & (4.7)\\ k_4 &= (E, \mathbf{0}, 0, -p, (-1)^r M_S) \equiv (K_4, (-1)^r M_S)\,, & (4.8) \end{aligned}$$

where $\mathbf{0}$ represents the zero momentum in the six dimensions transverse to the scattering plane, p is defined by

$$p = \frac{M_S v}{(1-v^2)^{1/2}}\,, \tag{4.9}$$

and

$$E^2 = \frac{M_S^2}{(1-v^2)} = E'^2\,. \tag{4.10}$$

The distinct choices $r = 0, 1$ and $0 \le l \le r$ allow for the three processes defined above.

The nine-dimensional Mandelstam variables, $S = -(K_1+K_2)^2$, $T = -(K_1+K_4)^2$ and $U = -(K_1+K_3)^2$ are given by

$$\begin{aligned} S &= -\frac{2M_S^2 v^2}{(1-v^2)}(1-\cos\theta)\,, \qquad T = 4M_S^2 + \frac{4M_S^2 v^2}{(1-v^2)}\,,\\ U &= -\frac{2M_S^2 v^2}{(1-v^2)}(1+\cos\theta)\,. \end{aligned} \tag{4.11}$$

These satisfy the mass-shell condition $S + T + U = 4M_S^2$.

The corresponding expressions for the ten-dimensional Mandelstam invariants, s, t, u, depend importantly on which of the three kinds of process is being considered. Given these identifications, the nine-dimensional amplitude can be obtained directly from the ten-dimensional expression (3.2).

(a) $l = 0$, $r = 0$: $D_S + D_S \to D_S + D_S$

In this case the ten-dimensional kinematic invariants are

$$s = S\,, \qquad t = T - 4M_S^2 = \frac{4M_S^2 v^2}{(1-v^2)}\,, \qquad u = U\,. \tag{4.12}$$

This is the process that is near-BPS at low energy. Substituting (4.12) into (3.2) gives an amplitude for D_S scattering proportional to

$$\frac{2M_S^2 v^2}{(1-v^2)}\left(2\frac{(1+\cos(\theta))}{(1-\cos(\theta))}(\epsilon_1\cdot\epsilon_2)(\epsilon_3\cdot\epsilon_4)+2(\epsilon_1\cdot\epsilon_3)(\epsilon_2\cdot\epsilon_4)-\right.$$
$$\left.-(1+\cos(\theta))(\epsilon_1\cdot\epsilon_4)(\epsilon_2\cdot\epsilon_3)\right), \tag{4.13}$$

where we have chosen polarisation vectors that satisfy $k_r\cdot\epsilon_s=0$ for all $r,s=1,\ldots,4$. This expression vanishes in the limit of zero velocity, $v\to 0$.

In this case the amplitude is closely related to the familiar type-IIA D-particle scattering amplitude studied in [15, 20]. In the string theory description the amplitude is defined in the impact parameter representation, where the back reaction of the interaction on the particle trajectories is ignored. This is valid if the D-particles are assumed to move in straight lines with slow relative velocity $2v$ and are separated by a transverse distance b. In the standard type-IIA theory the metric on the moduli space of two D0's is flat and the leading velocity-dependent term in the amplitude is of order v^4.

We are here concerned with the situation in which the two D-particles are constrained to lie on different fixed orientifold planes so the distance of separation is at least $\pi R_{11}l_p$. In order to relate the Feynman diagram we considered earlier to the standard D-particle results of [15] the amplitude must be expressed in terms of b and v instead of the Mandelstam variables S and T. This amounts to replacing T by its expression in terms of v using (4.12) and Fourier transforming with respect to the momentum transfer in the seven directions transverse to the particle trajectories. In the special frame defined by (4.5)–(4.8) this momentum transfer has been rotated into the vector $\mathbf{p}\sim(\mathbf{0},p\sin\theta)$ and $S\sim-|\mathbf{p}|^2$. Thus, the impact parameter representation is defined by

$$\tilde{A}_4(v,b)\sim\text{const.}\int dS\,(-S)^{5/2}\,A_4(S,T)\,e^{i\sqrt{-S}\,b}\,, \tag{4.14}$$

where the overall constant includes the volume of S^6 to account for the integration over angular components of the momentum transfer. The transverse separation of the two D_S particles is $|\pi R_{11}l_p+ib|$ and for $R_{11}\neq 0$ the integral in (4.14) is dominated by the region in which $0\ll\sqrt{-S}R_{11}l_p\ll M_S v$, where the factor of $1/\sinh(\pi\sqrt{-S}R_{11}l_p)$ in A_4 can be approximated by $2e^{-\pi\sqrt{-S}R_{11}l_p}$. Ignoring the deflection of the particles amounts to picking out the leading term in the $1/|\pi R_{11}l_p+ib|$ expansion of the amplitude and it is easy to see that this leading behaviour has a coefficient proportional to v^4 which comes from the factor of tu associated with the $\epsilon_1\cdot\epsilon_2\epsilon_3\cdot\epsilon_4$ term.

This behaviour is in agreement with the string calculation that can be performed as in [15]. In this case the relevant amplitudes consist of the sum of three kinds of world-sheets. The first diagram is the cylinder that describes the overlap between the two D-particle boundary states representing the two D_S's. The second kind of diagram consists of the Möbius strips that describes the overlap involving one D0 boundary state and the crosscap associated with the mirror image of that particle in the other orientifold plane. Finally, there are the cylinder diagrams that describe the overlap of either D-particle with any of the sixteen D8-brane boundary states. It is easy to argue that only the D0-D0 contribution can depend on v since the velocity is constrained to be parallel to the orientifold plane and D8. The D0-D0 contribution is therefore identical to the bulk term calculated in [15] and behaves as v^4 as expected.[4]

(b) $l = 0$, $r = 1$: $D_S + \bar{D}_S \to D_S + \bar{D}_S$

In this case the ten-dimensional kinematic invariants are given by

$$s = S\,, \qquad t = T\,, \qquad u = U - 4M_S^2 = -4M_S^2 - \frac{2M_S^2 v^2}{(1-v^2)}(1+\cos\theta)\,. \tag{4.15}$$

The non-BPS amplitude that results by substituting these expressions into (3.2) behaves to leading order in v as

$$\frac{8M_S^2(1-v^2)}{v^2(1-\cos(\theta))}(\epsilon_1\cdot\epsilon_2)(\epsilon_3\cdot\epsilon_4)\,, \tag{4.16}$$

which diverges as $v \to 0$. This divergence is presumably related to the instability of the system to decay into a non-BPS D-string [9, 10] which will not be discussed further here.

(c) $l = 1$, $r = 1$: $D_S + \bar{D}_S \to \bar{D}_S + D_S$

The ten-dimensional kinematic invariants are

$$s = S - 4M_S^2 = -4M_S^2 - \frac{2M_S^2 v^2}{(1-v^2)}(1-\cos\theta)\,, \qquad t = T\,, \qquad u = U\,. \tag{4.17}$$

This is the case in which there is D0 exchange and the behaviour of the amplitude is qualitatively different from the cases with $l = 0$ since $s \to -4M_S^2$ is large in the low-velocity limit, $v \to 0$. The denominator of (3.2) can then be expanded as

$$\frac{1}{\sinh(\pi\sqrt{-s}R_{11}l_p)} = 2e^{-2\pi M_S R_{11} l_p}\sum_{r=0}^{\infty} e^{-4\pi r M_S R_{11} l_p}\,. \tag{4.18}$$

Using the expression for M_S from (4.4), (4.18) then becomes

$$2\sum_{r=0}^{\infty} e^{-(2r+1)\pi R_{11}/R_9} = 2\sum_{r=0}^{\infty} e^{-(2r+1)\pi R^{\mathrm{IA}}/g_{\mathrm{IA}}} = 2\sum_{r=0}^{\infty} e^{-(2r+1)\pi/g_{\mathrm{I}}}\,, \tag{4.19}$$

where we have used the standard relation between the M-theory variables and the ten-dimensional type-IA coupling constant, $g_{\mathrm{IA}} = e^{\phi^{\mathrm{IA}}}$, and radius, R_{IA},

$$R_{\mathrm{IA}} = R_{11}R_9^{1/2}\,, \qquad g_{\mathrm{IA}} = R_9^{3/2}\,, \tag{4.20}$$

[4]The leading contribution in the case of a *bulk* D0 moving transverse to the orientifold planes is proportional to v^2. This reflects the reduced amount of supersymmetry in the system.

together with the familiar T-duality relation between type IA and type I,

$$g_{\rm I} = \frac{g_{\rm IA}}{R_{\rm IA}} = \frac{R_9}{R_{11}}\,, \qquad R_{\rm I} = \frac{1}{R_{\rm IA}} = \frac{1}{R_{11}R_9^{1/2}}\,. \tag{4.21}$$

In order to rewrite the total amplitude (3.2) in terms of type-IA (or type-I) variables, we also have to re-express the dimensionful quantities in terms of the string scale. As before, the amplitude of the original external states is proportional to $\kappa^{-2/3}$ which is now equal to

$$\kappa^{-2/3} = \frac{1}{(2\pi l_p)^3} = \frac{1}{(2\pi l_s^{\rm I})^3} e^{-\phi^{\rm IA}}\,. \tag{4.22}$$

Together with $\sqrt{-s} = 2M_S = (g_{\rm IA} l_s^{\rm I})^{-1}$ the amplitude thus becomes

$$A_4^{\rm I} = \frac{1}{(2\pi l_s^{\rm I})^2}\frac{1}{(4\pi)^{13/3}}\,{\rm Tr}(T_1T_2)\,{\rm Tr}(T_3T_4)t_8F^4\sum_{r=0}^{\infty} e^{-(2r+1)\pi/g_{\rm I}}\,. \tag{4.23}$$

This expansion as a series of exponentially suppressed terms is relevant at weak type-I coupling, $g_{\rm I} = R_9/R_{11} \ll 1$, where the leading term dominates. This term can be identified as the effect of a non-BPS D-instanton with action $\pi/g_{\rm I}$.[5] According to [11, 8] the type-I D-instanton can be thought of as the sum of the type-IIB D-instanton and the anti-D-instanton (without any prefactors). The orientation-reversing operator Ω maps the D-instanton boundary state to the anti-D-instanton boundary state, and therefore only the sum of the two boundary states is invariant. Under T-duality, the D-instanton becomes a D0-brane whose world-line stretches across the interval between the orientifold planes of the nine-dimensional type-IA theory, and the anti-D-instanton becomes the configuration in which the world-line has the opposite orientation. Again this can be represented by two boundary states that are mapped into one another under the T-dual of Ω, $\Omega\mathcal{I}_{11}$. The D-particle world-line transfers D0 Ramond-Ramond charge from one orientifold plane to the other, while the image under $\Omega\mathcal{I}_{11}$ transfers the opposite D0 Ramond-Ramond charge. In the process D0+$\bar{\rm D}$0 $\to$ $\bar{\rm D}$0+D0 only of these two processes contributes; the conjugate process $\bar{\rm D}0 + {\rm D}0 \to {\rm D}0 + \bar{\rm D}0$ would pick out the other contribution.

In our conventions, the mass of a bulk D-particle in type IA is $1/g_{\rm IA}$, and comparison with type IIA gives $g_{\rm IA} = g_{IIA}$. On the other hand, under T-duality, the stuck D-particle of type IA (with mass $1/(2g_{\rm IA})$) becomes a D1-brane of type I that wraps the circle $R_{\rm I}$ once. This has a mass $R_{\rm I}/(2g_{\rm I})$ and therefore $2g_{\rm I} = g_{\rm IIB}$. Using this, the action of the type-I D-instanton, that is the modulus of the exponent of the $r = 0$ term in (4.23), has the value

$$\frac{\pi}{g_{\rm I}} = \frac{2\pi}{g_{\rm IIB}}\,, \tag{4.24}$$

[5] As we shall demonstrate in the next section, this process also exhibits the key characteristic of the D-instanton of type I: it breaks O(32) to SO(32).

in type-IIB units. This is precisely the action of a single D-instanton of type IIB [16]. The agreement is a consequence of the BPS nature of the exchanged type-IIA D-particle.

The terms with $r > 0$ in (4.23) describe processes where the D-particle is emitted on one boundary and absorbed at the other, but where it winds an integer number of times around the compact circle. These terms appear as multi-instanton contributions with odd D-instanton number. However, all of these multi-instanton terms carry the same topological $\mathbb{Z}_2$ charge, and the contributions with $r > 0$ are thus presumably "unstable" and therefore only the leading contribution with $r = 0$ should be taken seriously in the perturbative type-I limit. However, the complete series is crucial in determining the amplitude in the limit $g_{\rm I} \to \infty$, which is the weak coupling limit of the SO(32) heterotic string theory (the HO theory) compactified on a circle with appropriate Wilson lines to break the symmetry to $\mathrm{SO}(16) \times \mathrm{SO}(16)$. The heterotic coupling is given by $g_{\rm HO} = 1/g_{\rm I}$ so that the amplitude (4.23) has an infinite power series expansion in heterotic string perturbation theory. After taking into account the rescaling of the string scale

$$l_s^{\rm I} = l_s^H \left(\frac{R_{11}}{R_9}\right)^{1/2} = l_s^H g_{\rm HO}^{1/2}\,, \tag{4.25}$$

needed to pass from the type I to the heterotic SO(32) theory, the amplitude has an expansion in powers of $g_{\rm HO}^2$,

$$A_4^{\rm HO} \sim \frac{1}{g_{\rm HO}^2 (2\pi l_s^H)^2} \operatorname{Tr}(T_1T_2) \operatorname{Tr}(T_3T_4) t_8 F^4 (1 + O(g_{\rm HO}^2))\,, \tag{4.26}$$

where we have used

$$\sum_{r=0}^{\infty} e^{\frac{-(2r+1)\pi}{g_{\rm I}}} = \frac{1}{2\sinh(\pi g_{\rm HO})} = \frac{1}{2\pi g_{\rm HO}}\Big(1 + O(g_{\rm HO}^2)\Big)\,. \tag{4.27}$$

The first term in (4.26) is a tree-level term that can be determined directly from the tree-level heterotic string compactified on a circle with the appropriate Wilson lines to give the unbroken $\mathrm{SO}(16)\times\mathrm{SO}(16)$. The $O(g_{\rm HO}^2)$ corrections appear to correspond to loop corrections to the amplitude, but these might also get contributions from M-theory loops that have not been considered in this paper.

The M-theory tree-level calculation can be trusted whenever both R_{11} and R_9 are large. In particular, this implies that the ten-dimensional type-IA coupling is large, and that the radius of the type-I theory is small. On the other hand, the ten-dimensional type-I coupling only depends on the ratio of R_9 and R_{11}, and therefore can be small. However this does not imply that one can trust the type-I perturbation. In fact, $R_{\rm I} = g_{\rm I}/g_{\rm IA} \ll g_{\rm I}$ and therefore the condensation of closed-string winding states cannot be ignored. This can also be seen from the fact that both the radius and the ten-dimensional coupling constant of the dual type-IA theory are large.

If we had been working in a regime in which perturbative string theory could be trusted there would have been a paradox. We would have been led to believe that the type-I D-instanton is only a "stable" solution (i.e. a solution with no tachyonic modes) provided the radius $R_{\rm I}$ is sufficiently large. Within the perturbative approximation, when $R_{\rm I} < \sqrt{2}$ the open string beginning and ending on the D-instanton develops a tachyonic mode and the D-instanton should "decay" into a non-BPS D0-$\bar{\rm D}$0 pair whose world-lines stretch along the circle.[6] Furthermore, these D0 and $\bar{\rm D}$0 branes carry a relative $\mathbb{Z}_2$ Wilson line. Under T-duality, the two $\mathbb{Z}_2$ D0 branes become two $\mathbb{Z}_2$ D-instantons of type IA, and the relative Wilson line means that they are located at opposite orientifold planes. From the point of view of type IA, the above superposition of D0 world-lines would therefore be unstable to decay into two non-BPS type-IA D-instantons provided that $R_{\rm IA} > 1/\sqrt{2}$. It is easy to see that such a configuration is not at all similar to the exchange of a D-particle that is contained in the M-theory amplitude.

However, the region of validity of our M-theory argument is one of strong coupling in the type-IA theory, in which the preceding stability arguments are not valid. The fact that the D-instanton action extracted from the spinor scattering process corresponds to that expected for large $R_{\rm I}$ in the type-I theory suggests that the non-BPS type-I D-instanton remains stable in the non-perturbative region $R_{\rm I} \ll g_{\rm I}$. The reason why the M-theory calculation reproduces the correct value for the instanton action is related to the fact that for the process we are considering, only one term in the superposition of the D-instanton and the anti-D-instanton contributes. The corresponding D-instanton action is therefore protected by supersymmetry since the bulk D0-brane that is exchanged is a BPS state of type IIA (and the anti-D0-brane does not play a role). We thus expect that the exponent in the amplitude can be trusted beyond the original regime of validity. Although the limit which gives ten-dimensional weakly coupled type-I theory reproduces the correct normalisation of the instanton action it will not give the correct value for the coefficient of the exponential.

4.2 The type-I D-instanton and the breaking of O(32)

From its description in terms of K-theory, it is clear that the D-instanton is associated with the fact that the gauge group of type-I string theory is SO(32) rather than the O(32) that might have been expected on the basis of perturbation theory [11].[7] Here we will demonstrate this by considering the dual type-IA theory in the standard SO(16)$\times$SO(16) vacuum. Consideration of the perturbative approximation to the low energy effective field theory would suggest that the symmetry group could be O(16)$\times$

[6]The K-theory class that corresponds to the D-instanton is however non-trivial for all values of the radii. The question of stability is the question of whether the representative of the non-trivial K-theory class that has least action is a pointlike instanton or an extended (one-dimensional) object.

[7]The actual gauge group is obviously Spin(32)/$\mathbb{Z}_2$. The issue of the correct spin cover will be discussed later.

O(16). However, we will see that the instantonic contribution to the scattering of spinor states is not invariant under the disconnected component of either of the two O(16) groups.

We shall concentrate on the O(16) group that is associated to the left orientifold plane since the argument for the other gauge group is identical. In the vacuum we are considering, eight D8-branes (plus eight mirror D8-branes) are located at each of the two orientifold planes. As we have seen, the instantonic contribution to the scattering amplitude comes from the diagram where a D0 (or $\bar{\text{D}}0$) is emitted from the left orientifold plane and absorbed at the right orientifold plane. In this process, the D0 (and its mirror partner) has to cross the eight D8-branes (and their mirror partners) that are localised at the fixed plane. However, whenever a D0 crosses a D8-brane, a fundamental string is created that stretches between the D0 and the D8-brane [23]. This is an important effect as we shall show momentarily.

In order to obtain a clear picture, it is useful to consider the configuration where the eight D8-branes have been moved to $x^{11} = \epsilon$, away from the orientifold fixed plane at $x^{11} = 0$ (with corresponding displacements of the mirror eight-branes to $x^{11} = -\epsilon$). In this configuration, the "cosmological constant" m takes the value $m = -8$ for $0 < x^{11} < \epsilon$ and $m = 0$ for $|x^{11}| > \epsilon$. As was shown by [24], in a background characterised by m, each D0 has to carry $|m|$ strings, where the sign of m determines whether the string begins or ends on the D0. The D0 (and its mirror) that are emitted from the orientifold plane each carry eight strings in the regime $|x^{11}| < \epsilon$. These strings can be thought of as stretching between the bulk D0 (that has been emitted from the orientifold plane) and the remaining fractional D0 that is still stuck. As the bulk D0 (and its mirror) crosses the eight D8-branes (and their mirrors), additional strings are created that join it to the D8-branes. These can recombine with the original strings to give eight strings that stretch between the stuck D0 and the eight D8-branes, as well as eight strings that stretch between the stuck D0 and the eight mirror D8-branes. In the final configuration the bulk D0 does not have any strings attached to it, which is consistent with the fact that $m = 0$ in the interval between the two fixed planes.

The sixteen different strings that stretch between the stuck D0 and the eight plus eight D8-branes can be identified with the generators γ_i, $i = 1, \ldots, 16$ of the Clifford algebra

$$\{\gamma_i, \gamma_j\} = 2\delta_{ij} \tag{4.28}$$

that give rise to the Lie algebra elements of so(16) by

$$\sigma_{ij} = \frac{1}{4}[\gamma_i, \gamma_j]\,. \tag{4.29}$$

Indeed, the elements of the gauge group correspond to 8-8 strings, and are therefore bilinear in the 0-8 strings that are described by the γ_i. The product of all sixteen 0-8 strings (that arises naturally in the above configuration) is then the *chirality operator* of O(16),

$$\Gamma = \prod_{i=1}^{16} \gamma_i \,. \tag{4.30}$$

Thus it follows that the vertex that describes the emission of a single D0 (or $\bar{\mathrm{D}}$0) is actually given by

$$\mathrm{Tr}\,(S_1 S_2 \Gamma)\,, \tag{4.31}$$

where S_i describes the state in the spinor representation for the two fractional D0's. More generally, if m bulk D0's are emitted, the vertex will be

$$\mathrm{Tr}\,(S_1 S_2 \Gamma^m)\,, \tag{4.32}$$

since each bulk D0 gives rise to one chirality operator.

The chirality operator Γ is invariant under conjugation by any element of SO(16), but it changes sign under conjugation by an element g_0 in the disconnected component of O(16), $g_0 \Gamma g_0^{-1} = -\Gamma$. This implies that the vertex with odd m only respects the SO(16) subgroup of O(16). Since the contributions with odd m are precisely those that appear in the instantonic contribution to the amplitude, this exponentially suppressed contribution is associated with the breaking of O(16) to SO(16). In terms of type I, SO(16) $\times$ SO(16) is a subgroup of SO(32). The large O(32) transformations that are not in SO(32) can be taken to lie in O(16) $\times$ SO(16); these transformations are indeed not respected by the D-instanton, as was stressed in [11].

The actual gauge group of the ten-dimensional type-I theory is Spin(32)/$\mathbb{Z}_2$. Upon compactification to nine dimensions with the above Wilson line, this is broken to S(Pin(16) $\times$ Pin(16))/$\mathbb{Z}_2$, where the S outside the bracket indicates that either both the elements of Pin(16) $\times$ Pin(16) are in the disconnected component of Pin(16) or neither of them are. As we have just explained, the effect of the D-instanton of type I is to impose this constraint. The actual gauge group is somewhat smaller since the nine-dimensional type-IA theory also has a D-instanton that is T-dual to the wrapped D0-brane of type I. This instanton is stuck on one of the two fixed planes, and it is responsible for breaking each Pin(16) separately to Spin(16). (This follows by essentially the same arguments that were used by Witten for the case of the ten-dimensional type-I D-instanton.) Thus the actual gauge group is

$$\left(\mathrm{Spin}(16) \times \mathrm{Spin}(16)\right)/\mathbb{Z}_2\,, \tag{4.33}$$

where the element in the centre by which the group is divided is the element that does not allow states transforming in the vector representation of one Spin(16) and the scalar representation of the other. It is not difficult to see that all allowed (and no other) conjugacy classes of representations of this gauge group are actually present in both nine dimensional heterotic theories. (This discussion is relevant for some issues raised in [25].)

5. Discussion

In this paper we have identified the D-instanton contribution to a certain amplitude involving spinor states of type I, using the relation of nine-dimensional type I to M-theory. This relation involves type IA, and it is therefore natural to ask whether the $\mathbb{Z}_2$ non-BPS D-instanton of type IA (that is T-dual to the D0-brane of type I with its world-line wrapped along x^9) can also be understood in terms of M-theory (compare also [26]). The wrapped D0-brane of type I can be obtained from a D1-brane anti-D1-brane pair that are wrapped around x^9 and x^8 say, and that have a relative Wilson line along x^8. Under T-duality of the x^9-circle, we therefore obtain a D0-brane and an anti-D0-brane, both localised on the *same* orientifold plane and wrapped around x^8 with a relative Wilson line. One should therefore expect that the type-IA D-instanton contributes to the scattering diagram $D_S + \bar{D}_S \to \bar{D}_S + D_S$ where now all four D-particles are localised on the same boundary. An analysis of this process can be made that is analogous to that of the main part of the paper, with the important difference that the propagator (2.4) does not involve a $(-1)^m$, and therefore becomes

$$\widehat{\mathcal{G}}(p^M;0,0) = \frac{1}{\pi R_{11} l_p} \sum_{m=-\infty}^{\infty} \frac{1}{p^2 + \frac{m^2}{R_{11}^2 l_p^2}} = \frac{1}{\sqrt{p^2}} \sum_{n\in\mathbb{Z}} e^{-2|n|\pi\sqrt{p^2}R_{11}l_p} . \qquad (5.1)$$

The term with $n = 0$ should correspond to a part of the contribution of the type-IA $\mathbb{Z}_2$ D-instanton. The terms with $n \neq 0$ come from multiple windings of the D0 world-line and, as in the earlier case, carry the same $\mathbb{Z}_2$ topological charge as the $n = 0$ term; the latter is therefore the dominant contribution. Thus the tree amplitude describing the exchange of the graviton and the three-form in type-IA variables is approximated by

$$A_4 = \frac{1}{2(2\pi l_s^{\mathrm{I}})^2} \frac{1}{(4\pi)^{13/3}} t_8 F^4 \Big[\mathrm{Tr}(T_1T_2)\,\mathrm{Tr}(T_3T_4) + \cdots \Big] , \qquad (5.2)$$

where we have only exhibited explicitly the term whose group structure is the same as in the main part of the paper. By essentially the same arguments as above, the coupling of the spinors to the bulk graviton and three-form should again involve a chirality operator and therefore break O(16) to SO(16). However, the type-IA instanton should contribute a term of the form $e^{-c/g_{\mathrm{IA}}}$ to the amplitude (for some constant, c). Since the M-theory calculation is justified only in the region of large large R_9, R_{11}, where $g_{\mathrm{IA}} \gg 1$, the expression (5.2) can only represent the first term in an expansion of the exponent in powers of $1/g_{\mathrm{IA}}$. The higher-order terms will depend on the undetermined loop diagrams of M-theory.

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A. The Hořava-Witten action

The spinor indices are written as α, β, γ. The supergravity multiplet has the graviton g, the gravitino $\psi_{\mu\alpha}$, and a three-form C with the field strength G (in component form $G_{\mu\nu\rho\sigma} = 4!\partial_{[\mu}C_{\nu\rho\sigma]}$). The spinors are Majorana and $\overline{\psi}_\alpha$ is defined by $\overline{\psi}_\alpha = C_{\alpha\beta}\psi^\beta$ where $C_{\alpha\beta}$ is the charge conjugation matrix. The 32×32 real Dirac matrices Γ_μ satisfy the Clifford algebra $\{\Gamma_\mu, \Gamma_\nu\} = 2g_{\mu\nu}$, and we define $\Gamma^{\mu_1\mu_2\cdots\mu_n} \equiv \frac{1}{n!}\Gamma^{\mu_1}\Gamma^{\mu_2}\cdots\Gamma^{\mu_n}\pm$ permutations. With these conventions the $D = 11$, $N = 1$ bulk supergravity action is given by [19],

$$\begin{aligned} S_{\text{bulk}} = \frac{1}{\kappa^2}\int_{\mathbb{R}^{1,9}\times S^1} d^{11}x\sqrt{-g}\Big\{ &-\frac{R}{2} - \frac{1}{2}\overline{\psi}_\mu\Gamma^{\mu\nu\rho}D_\nu\psi_\rho - \frac{1}{48}G_{\mu\nu\rho\sigma}G^{\mu\nu\rho\sigma} - \\ &-\frac{\sqrt{2}}{192}\left(\overline{\psi}_\mu\Gamma^{\mu\nu\rho\sigma\tau\lambda}\psi_\lambda + 12\overline{\psi}^\nu\Gamma^{\rho\sigma}\psi^\tau\right)G_{\nu\rho\sigma\tau} - \\ &-\frac{\sqrt{2}}{3456}\epsilon^{\mu_1\mu_2\cdots\mu_{11}}C_{\mu_1\mu_2\mu_3}G_{\mu_4\cdots\mu_7}G_{\mu_8\cdots\mu_{11}}\Big\}. \end{aligned} \tag{A.1}$$

Here $\kappa^2 = (2\pi l_p)^9$, where l_p is the eleven-dimensional Planck length. We are not considering higher order terms quartic in the gravitino. The Riemann tensor is the field strength of the spin connection Ω. On the other hand the $D = 10$ supersymmetric Yang-Mills action on each boundary, containing the E_8 gauge field A^a and the gluino χ^a, coupled to the bulk supergravity fields, is given by [14],

$$\begin{aligned} S_{YM} = &\frac{1}{(4\pi)^{5/3}\kappa^{4/3}}\int_{R^{1,9}} d^{10}x\sqrt{-g}\,\times \\ &\times\Big[-\frac{1}{4}F^a_{MN}F^{aMN} - \frac{1}{2}\bar{\chi}^a\Gamma^M D_M\chi^a - \frac{1}{4}\bar{\psi}_M\Gamma^{NP}\Gamma^M F^a_{NP}\chi^a + \bar{\chi}^a\Gamma^{MNP}\chi^a\,\times \\ &\quad\times\Big\{\frac{\sqrt{2}}{48}G_{MNP11} + \frac{1}{32}\bar{\psi}_M\Gamma_{NP}\psi_{11} + \frac{1}{32}\bar{\psi}^D\Gamma_{DMNP}\psi_{11} + \\ &\qquad + \frac{1}{128}\Big(3\bar{\psi}_M\Gamma_N\psi_P - \bar{\psi}_M\Gamma_{NPQ}\psi^Q - \frac{1}{2}\bar{\psi}_Q\Gamma_{MNP}\psi^Q - \\ &\qquad\qquad - \frac{13}{6}\bar{\psi}^D\Gamma_{QMNPR}\psi^R\Big)\Big\}\Big], \end{aligned} \tag{A.2}$$

where g is here the restriction of the eleven-dimensional metric to ten dimensions, and the Yang-Mills field strength is given by $F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M + f^a_{bc}A^b_M A^c_N$. The Yang-Mills coupling λ has been expressed in terms of the gravitational coupling κ by the relation $\lambda^2 = 4\pi(4\pi\kappa^2)^{2/3}$ [14]; the prefactor is therefore $(4\pi)^{5/3}(2\pi l_p)^6$. The expression for G_{MNP11} is modified in the manner described by (2.10).

B. The separate tree amplitudes

The tree amplitude due to graviton exchange is given by

$$
\begin{aligned}
A_4^{(h)} = {} & \frac{\kappa^{-2/3}}{(4\pi)^{10/3} d} \operatorname{Tr}(T_1 T_2) \operatorname{Tr}(T_3 T_4) \sum_{m \in \mathbb{Z}} (-1)^m \frac{1}{-s + p_{11}^2} \times \\
& \times \left[\epsilon_1^{N_1} k_1^{M_1} \epsilon_2^{Q_1} k_2^{P_1} + \epsilon_1^{Q_1} k_1^{P_1} \epsilon_2^{N_1} k_2^{M_1} \right] \times \\
& \times \left(\frac{1}{2} \eta_{M_1[Q_1} \eta_{P_1]N_1} \eta_{R_1 S_1} + \eta_{M_1[P_1} \eta_{Q_1]S_1} \eta_{R_1 N_1} + \eta_{M_1[R_1} \eta_{Q_1]N_1} \eta_{P_1 S_1} \right) \times \\
& \times \left(\eta^{R_1 R_2} \eta^{S_1 S_2} + \eta^{R_1 S_2} \eta^{S_1 R_2} - \frac{2}{9} \eta^{R_1 S_1} \eta^{R_2 S_2} \right) \times \\
& \times \left(\frac{1}{2} \eta_{M_2[Q_2} \eta_{P_2]N_2} \eta_{R_2 S_2} + \eta_{M_2[P_2} \eta_{Q_2]S_2} \eta_{R_2 N_2} + \eta_{M_2[R_2} \eta_{Q_2]N_2} \eta_{P_2 S_2} \right) \times \\
& \times \left[\epsilon_3^{N_2} k_3^{M_2} \epsilon_4^{Q_2} k_4^{P_2} + \epsilon_3^{Q_2} k_3^{P_2} \epsilon_4^{N_2} k_4^{M_2} \right] .
\end{aligned} \tag{B.1}
$$

The sum over m can be performed directly using (2.5), and after a lengthy calculation the result simplifies to

$$
\begin{aligned}
A_4^{(h)} = {} & \frac{\kappa^{-2/3}}{4(4\pi)^{10/3}} \operatorname{Tr}(T_1 T_2) \operatorname{Tr}(T_3 T_4) \frac{1}{\sqrt{-s}} \frac{1}{\sinh(\sqrt{-s} d)} \times \\
& \times \Big\{ -2ut(\epsilon_1 \cdot \epsilon_2)(\epsilon_3 \cdot \epsilon_4) + s^2 (\epsilon_1 \cdot \epsilon_3)(\epsilon_2 \cdot \epsilon_4) + s^2 (\epsilon_1 \cdot \epsilon_4)(\epsilon_2 \cdot \epsilon_3) + \\
& \quad + (\epsilon_1 \cdot \epsilon_2) \Big[4t(\epsilon_3 \cdot k_1)(\epsilon_4 \cdot k_2) + 4u(\epsilon_3 \cdot k_2)(\epsilon_4 \cdot k_1) \Big] + \\
& \quad + (\epsilon_3 \cdot \epsilon_4) \Big[4t(\epsilon_1 \cdot k_3)(\epsilon_2 \cdot k_4) + 4u(\epsilon_1 \cdot k_4)(\epsilon_2 \cdot k_3) \Big] + \\
& \quad + (\epsilon_2 \cdot \epsilon_3) \Big[+2s(\epsilon_1 \cdot k_4)(\epsilon_4 \cdot k_3) + 2s(\epsilon_1 \cdot k_2)(\epsilon_4 \cdot k_1) - 2t(\epsilon_1 \cdot k_2)(\epsilon_4 \cdot k_3) \Big] + \\
& \quad + (\epsilon_1 \cdot \epsilon_3) \Big[+2s(\epsilon_2 \cdot k_4)(\epsilon_4 \cdot k_3) + 2s(\epsilon_2 \cdot k_1)(\epsilon_4 \cdot k_2) - 2u(\epsilon_2 \cdot k_1)(\epsilon_4 \cdot k_3) \Big] + \\
& \quad + (\epsilon_2 \cdot \epsilon_4) \Big[+2s(\epsilon_1 \cdot k_3)(\epsilon_3 \cdot k_4) + 2s(\epsilon_1 \cdot k_2)(\epsilon_3 \cdot k_1) - 2u(\epsilon_1 \cdot k_2)(\epsilon_3 \cdot k_4) \Big] + \\
& \quad + (\epsilon_1 \cdot \epsilon_4) \Big[+2s(\epsilon_2 \cdot k_3)(\epsilon_3 \cdot k_4) + 2s(\epsilon_2 \cdot k_1)(\epsilon_3 \cdot k_2) - 2t(\epsilon_2 \cdot k_1)(\epsilon_3 \cdot k_4) \Big] - \\
& \quad - 4(\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1)(\epsilon_3 \cdot k_4)(\epsilon_4 \cdot k_3) + 4(\epsilon_1 \cdot k_4)(\epsilon_2 \cdot k_1)(\epsilon_3 \cdot k_2)(\epsilon_4 \cdot k_3) + \\
& \quad + 4(\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_4)(\epsilon_3 \cdot k_1)(\epsilon_4 \cdot k_3) + 4(\epsilon_1 \cdot k_3)(\epsilon_2 \cdot k_1)(\epsilon_3 \cdot k_4)(\epsilon_4 \cdot k_2) + \\
& \quad + 4(\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_3)(\epsilon_3 \cdot k_4)(\epsilon_4 \cdot k_1) \Big\} .
\end{aligned} \tag{B.2}
$$

The three-form exchange diagram involves a vertex of the form S^{AAC} (2.12) at each end of the propagator, and thus the total amplitude is of the form

$$
\begin{aligned}
A_4^{(C)} &= \frac{2\kappa^{-2/3}}{(4\pi)^{10/3}(3!)^2 d} \operatorname{Tr}(T_1T_2)\operatorname{Tr}(T_3T_4)\sum_{m\in\mathbb{Z}}(-1)^m \frac{1}{-s+p_{11}^2}\times\\
&\quad\times\left[\epsilon_1^{M_1}k_2^{N_1}\epsilon_2^{P_1}+\epsilon_2^{M_1}k_1^{N_1}\epsilon_1^{P_1}\right]\times\\
&\quad\times\left(p^{[M_2}p_{[M_1}\eta_{N_1P_1}\eta_{11]}^{N_2}\eta^{P_2\,11]}\right)\left[\epsilon_3^{M_2}k_4^{N_2}\epsilon_4^{P_2}+\epsilon_4^{M_2}k_3^{N_2}\epsilon_3^{P_2}\right],
\end{aligned}
\tag{B.3}
$$

where $p=(k_1+k_2,p_{11})$, since the external gauge bosons do not have any momentum (or polarisation vectors) in the 11-th direction. The propagator therefore has two contributions

$$
\left(3k_{[M_2}k^{[M_1}\eta^{N_1P_1]}\eta_{N_2P_2]}-p_{11}^2\eta^{[M_1N_1}\eta_{[M_2}^{P_1]}\eta_{N_2P_2]}\right),
\tag{B.4}
$$

where $k=k_1+k_2=k_3+k_4$. In the first term we can again do the sum over p_{11} as before, and we obtain a contribution of the form

$$
\begin{aligned}
A_4^{(C)\,1} &= \frac{\kappa^{-2/3}}{4(4\pi)^{10/3}}\operatorname{Tr}(T_1T_2)\operatorname{Tr}(T_3T_4)\frac{1}{\sqrt{-s}}\frac{1}{\sinh(\sqrt{-s}d)}\times\\
&\quad\times\Big\{+(u-t)(\epsilon_1\cdot\epsilon_4)(\epsilon_2\cdot k_1)(\epsilon_3\cdot k_4)+(t-u)(\epsilon_1\cdot\epsilon_3)(\epsilon_2\cdot k_1)(\epsilon_4\cdot k_3)+\\
&\qquad+(t-u)(\epsilon_2\cdot\epsilon_4)(\epsilon_1\cdot k_2)(\epsilon_3\cdot k_4)+(u-t)(\epsilon_2\cdot\epsilon_3)(\epsilon_1\cdot k_2)(\epsilon_4\cdot k_3)+\\
&\qquad+(\epsilon_1\cdot k_3)(\epsilon_2\cdot k_1)(\epsilon_3\cdot k_4)(\epsilon_4\cdot k_1)-(\epsilon_1\cdot k_3)(\epsilon_2\cdot k_1)(\epsilon_3\cdot k_4)(\epsilon_4\cdot k_2)-\\
&\qquad-(\epsilon_1\cdot k_4)(\epsilon_2\cdot k_1)(\epsilon_3\cdot k_4)(\epsilon_4\cdot k_1)+(\epsilon_1\cdot k_4)(\epsilon_2\cdot k_1)(\epsilon_3\cdot k_4)(\epsilon_4\cdot k_2)+\\
&\qquad+(\epsilon_1\cdot k_4)(\epsilon_2\cdot k_1)(\epsilon_3\cdot k_1)(\epsilon_4\cdot k_3)-(\epsilon_1\cdot k_4)(\epsilon_2\cdot k_1)(\epsilon_3\cdot k_2)(\epsilon_4\cdot k_3)-\\
&\qquad-(\epsilon_1\cdot k_3)(\epsilon_2\cdot k_1)(\epsilon_3\cdot k_1)(\epsilon_4\cdot k_3)+(\epsilon_1\cdot k_3)(\epsilon_2\cdot k_1)(\epsilon_3\cdot k_2)(\epsilon_4\cdot k_3)+\\
&\qquad+(\epsilon_1\cdot k_2)(\epsilon_2\cdot k_3)(\epsilon_3\cdot k_4)(\epsilon_4\cdot k_2)-(\epsilon_1\cdot k_2)(\epsilon_2\cdot k_3)(\epsilon_3\cdot k_4)(\epsilon_4\cdot k_1)-\\
&\qquad-(\epsilon_1\cdot k_2)(\epsilon_2\cdot k_4)(\epsilon_3\cdot k_4)(\epsilon_4\cdot k_2)+(\epsilon_1\cdot k_2)(\epsilon_2\cdot k_4)(\epsilon_3\cdot k_4)(\epsilon_4\cdot k_1)+\\
&\qquad+(\epsilon_1\cdot k_2)(\epsilon_2\cdot k_4)(\epsilon_3\cdot k_2)(\epsilon_4\cdot k_3)-(\epsilon_1\cdot k_2)(\epsilon_2\cdot k_4)(\epsilon_3\cdot k_1)(\epsilon_4\cdot k_3)-\\
&\qquad-(\epsilon_1\cdot k_2)(\epsilon_2\cdot k_3)(\epsilon_3\cdot k_2)(\epsilon_4\cdot k_3)+(\epsilon_1\cdot k_2)(\epsilon_2\cdot k_3)(\epsilon_3\cdot k_1)(\epsilon_4\cdot k_3)\Big\}.
\end{aligned}
\tag{B.5}
$$

In the second term, the sum over m is evaluated using (2.6), and we obtain

$$
\begin{aligned}
A_4^{(C)\,2} &= \frac{\kappa^{-2/3}}{4(4\pi)^{10/3}}\operatorname{Tr}(T_1T_2)\operatorname{Tr}(T_3T_4)\sqrt{-s}\frac{1}{\sinh(\sqrt{-s}d)}\times\\
&\quad\times\Big\{+(t-u)(\epsilon_1\cdot\epsilon_3)(\epsilon_2\cdot\epsilon_4)+(u-t)(\epsilon_1\cdot\epsilon_4)(\epsilon_2\cdot\epsilon_3)+\\
&\qquad+(\epsilon_1\cdot\epsilon_3)\Big[(\epsilon_2\cdot k_3)-(\epsilon_2\cdot k_4)\Big]\Big[(\epsilon_4\cdot k_2)-(\epsilon_4\cdot k_1)\Big]+\\
&\qquad+(\epsilon_1\cdot\epsilon_4)\Big[(\epsilon_2\cdot k_3)-(\epsilon_2\cdot k_4)\Big]\Big[(\epsilon_3\cdot k_1)-(\epsilon_3\cdot k_2)\Big]+\\
&\qquad+(\epsilon_2\cdot\epsilon_3)\Big[(\epsilon_1\cdot k_3)-(\epsilon_1\cdot k_4)\Big]\Big[(\epsilon_4\cdot k_1)-(\epsilon_4\cdot k_2)\Big]-\\
&\qquad-(\epsilon_2\cdot\epsilon_4)\Big[(\epsilon_1\cdot k_3)-(\epsilon_1\cdot k_4)\Big]\Big[(\epsilon_3\cdot k_1)-(\epsilon_3\cdot k_2)\Big]\Big\}.
\end{aligned}
\tag{B.6}
$$

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